Math 10A with Professor Stankova
Quiz 14; Wednesday, 11/29/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** Let X be the number of heads I flip after flipping one coin. By the Central Limit Theorem, the average of X for 10 coin flips will be normally distributed.

Solution: The distribution will approach a normal distribution as $n \to \infty$ but will not be for finite n.

2. True **FALSE** By the Law of Large Numbers, if we take our sample size to be very large, the sample average will be equal μ, the population average, with high probability.

Solution: By the Law of Large Numbers, with high probability the sample average will be very close to μ. Remember that in a continuous distribution, the probability of getting an exact value is 0.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Suppose that 90% of students will pass MATH 10A. Let X be the random variable which takes in a student and outputs 1 if the student passes, and 0 otherwise.
   (a) (2 points) Calculate $E[X]$ and $SE(X)$.

Solution: $E[X] = 0.9 \cdot 1 + 0.1 \cdot 0 = 0.9$. Then $Var(X) = 0.9(1-0.9)^2 + 0.1(0-0.9)^2 = 0.09$. So $SE(X) = \sqrt{0.09} = 0.3$.

   (b) (4 points) Assuming students independently distributed, what is the probability that everyone in a class of 25 students passes the class (Hint: Do not use CLT)? You do not need to simplify your answer.

Solution: The probability of one student passing is 0.9 so the probability of 25 independently distributed students passing is $(0.9)^{25}$.

(c) (4 points) Using CLT, approximate the probability that in a class of 25 students, at least 96% of the class passes.

Solution: We are asking for $P(\bar{X} \geq 0.96)$. In a class of 25, we know that $\bar{X}$ is approximately normally distributed with an average of 0.9 and standard deviation $0.3/\sqrt{25} = 0.06$. So $P(X \geq 0.96) = 0.5 - z(|0.96 - 0.9|/0.06) = 0.5 - z(1)$.